Dijkstra’s Shortest Distance Connecting Lebanese Urban Centers

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Abstract

A network is a system of interconnected elements affecting communication, transportation, and energy flow (electricity …). Networks can be embodied as a set of nodes representing spatial locations, and a set of links representing connections. The Dijkstra’s algorithm finds the shortest path between two or more points. Paths with nodes not corresponding to intersections are usually used to map the connectivity of a network where an interference may occur between its different intersecting elements. Such maps were produced for Lebanese urban centers at both the scale of the whole Lebanon, and that of each of its mouhafazat (provinces). The chosen grid dimensions were optimized, having been minimized to entirely avoid path intersections, while simultaneously maintaining a sufficient size to facilitate feasible computational processing times. Summary statistics (average distance …) were inferred for the resulting networks with shortest paths. The Mount Lebanon mouhafaza shows the longest total shortest path, and that of Baalbek – Hermel the sparsest clustering of urban centers. The produced maps of shortest path constitute a fundamental basis for the development of any interference network type in Lebanon, especially considering the large development opportunities in the country.

Keywords: Cold Spot, Dijkstra’s Algorithm, Greater Beirut Area, Lebanon, Network Analyst, Grid Size, Shortest Path, Traveling Salesman Problem (TSP).

Introduction

Traveling occurs across networks. Cars and trucks run on roads, trains on railways, and airplanes may fly on pre-defined routes. Networks also affect communication (distance vector routing on the internet

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…). A network is a system of interconnected elements, such as lines connecting points. Networks can be embodied as a set of nodes representing spatial locations, and a set of links representing connections (see Xie & Levinson, 2007 for details). By modeling potential travel paths within a network, it is possible to analyze its traffic. Finding the shortest path between two or more points for a given network minimizes the use of the network. For example, the shortest path analysis on the roads is a useful tool to minimize the energy consumption of the vehicles, leading to a decrease in the level of pollution. Other examples include the building of the power grid. Self-driving cars are "trending". The implementation of the shortest path not only leads to a reduction in energy consumption but also serves to diminish the duration passengers must wait within a vehicle.

Network elements, such as edges (lines) and junctions (points), must be interconnected to allow for navigation over the network. These connections were first manually identified to attempt finding the shortest path, *e.g.*, by following straight lines. Solutions represented by connecting points following shortest path straight lines were applied for example to 24,978 cities, towns, and villages in Sweden (see Pavlus, 2013, p. 73). No shortest path network was so far determined for the Lebanese territory. Some major international applications (*e.g.*, Google Maps®, OSM And® …) rely on automatically finding the shortest path only between two points, not therefore permitting to pass by every single locality present in the neighborhood. If the search procedure, using these applications, was repeated to select more than two points, intersections of paths may occur. On the other hand, these applications follow the roads (useful mainly for vehicles) during the computation of the shortest distance. However, different network types require rather a straight line (*e.g.*, with the use of drones).

When the total number of points to be connected becomes large, the computation of the best solution using a deterministic approach becomes tedious even for the most powerful computers, hence the use of heuristic methods. Several heuristic search strategies have been developed for increasing the computational efficiency of shortest path search (Fu *et al.*, 2005), including the Traveling Salesman Problem (TSP). A network with path intersections either leads to a traffic jam at the intersections or changes the signal "concentration" and/or interference. As an example of the former, there would be a need for drone navigation control in case of path intersections. Examples of the latter include a drone spreading chemical substances at a constant rate in a field where there would be an increase of the concentration of the spread material at path intersections. Other examples of the latter include the mixing of substance concentrations in pipelines.

In simple terms, the Dijkstra algorithm (see Dijkstra, 1959) is used to find the shortest path between different points (nodes). It selects a novel node characterized by the minimal distance from the initial node, subsequently calculating the cumulative distance, accounting for traversal through this newly chosen node, in relation to each adjacent node. If the value of the sum of distances (by passing through three nodes - including the starting node) turns out to be smaller than that by any other path, the algorithm adds this third node to the general selected path.

The procedure is similarly repeated to find the successive nodes. The final path goes through all the points using the shortest distance. The present application of the Dijkstra's algorithm was performed using the Network Analyst tool, from ArcGIS®, to solve the shortest path problem.

A review of the path problems in networks and their algorithms is found in Baras and Theodorakopoulos (2010), and references therein. The shortest path problem is essential in communication networks. The model for shortest path routings is a graph, and the objects are the paths between two given vertices of that graph. Historically, the path problem has been dealt with using different path problem methodologies (classical, algebraic …). The focus in this paper is on the classical path problem. It combines graph theory and the shortest path problem.

The aim of this paper is to apply the Dijkstra's algorithm to connect all the Lebanese villages and towns together, without path intersection, using the shortest path.

Materials and methods

Shortest path algorithms

To compute the shortest path, three main algorithms - Dijkstra, Bellman-Ford, and Floyd-Warshall – are usually used. Each algorithm iterates on a different increasing set of paths. Distance vector routing on the internet, for example, uses the distributed version of Bellman-Ford, whereas link state routing uses Dijkstra's algorithm. The Bellman-Ford algorithm (see Bellman, 1958, and Lestor *et al.*, 1962) solves the Single Source Shortest Path (SSSP) even when there are negative weight edges. It iterates over the number of edges in the path. On the other hand, the Floyd-Warshall algorithm computes the shortest paths between all pairs of vertices (APSP - All-Pairs Shortest Paths) as opposed to just one source and all destinations. It iterates over increasing sets of vertices allowed to be used in the paths. Dijkstra's algorithm solves the SSSP when all edges have non-negative weights. It iterates over progressively longer path weights (Dijkstra, 1959). The path ends, in any map, were arbitrarily chosen by selecting 2 urban centers that are quite close to each other.

Grid size

The Dijkstra's algorithm was adapted to the current problem by creating a grid with cells of 100x100 m. The use of the grid was necessary, as some (small) villages in Lebanon have only one way that is used to both enter and exit the same village (back and forth travel along the same road). The road network's localized deficiencies necessitated the incorporation of intersections, as each stop could only be traversed once. Thus, straight line connections were used. By contrast, in large towns (*e.g.*, Greater Beirut), as the road network is dense enough (always more than one way to enter or exit any locality), the actual road infrastructure network was used instead of the grid. The network junctions represent road intersections, and the edges connect the junctions.

Dijkstra's algorithm for shortest path without intersection

In GIS, networks are widely used for two kinds of modeling - transportation and utility. The routing solvers within the ArcGIS® Network Analyst extension - namely Route, Closest Facility, and OD Cost Matrix solvers - are based on the Dijkstra's algorithm for finding shortest paths (see the section "Algorithms used by the ArcGIS® Network Analyst extension" in the User's Manual of the ArcGIS® Help Library). The classic Dijkstra's algorithm solves the single-source shortest-path problem on a weighted graph (Baras and Theodorakopoulos, 2010). Solving a route analysis is equivalent to finding the shortest route depending on the impedance chosen. Hence, the best route can be defined as the route that has the lowest impedance, or the least cost (see the section "Route analysis").

One map was produced for the Greater Beirut Area (GBA); one for each mouhafaza (plural mouhafazat) – including the capital Beirut; and the last for the whole Lebanon. For the latter, the path starts at Ras-Beyrouth (in Beirut), passes by all the Lebanese towns and villages, each once, and then returns to Ain el-Mreissé (in Beirut), without any path intersection. Each stop represents the center of a town or a village.

Source of polygon shape, location, and center

The borders of the Lebanese mouhafazat, together with those of its towns and villages, were obtained from the public database GADM (Global Administrative Areas) data, version 3.6, released on May 6th, 2018 (https://gadm.org), as a polygon Shapefile extension (shp). The boundaries of any selected urban center are delineated by the sides of each polygon. A line connecting any two urban centers has a starting point in the center of the first polygon, and an ending point in the center of the second.

Polygon coordinate determination method

The utilization of the most recent iteration of the World Geodetic System (WGS 84) in this context is attributed to its status as a cartographic, geodetic, and satellite navigation standard, encompassing Global Positioning System (GPS) applications. It includes the definition of the coordinate system's fundamental and derived constants, the ellipsoidal (normal) Earth Gravitational Model (EGM), a description of the associated World Magnetic Model (WMM), and a current list of local datum transformations.

Route analysis tool

Route analysis: Solving a route analysis can mean finding the quickest, shortest, or even the most scenic route, depending on the impedance chosen to solve for. If the impedance is time, then the best route is the quickest route. The best route can be defined as the route that has the lowest impedance. Finding the best route through a series of stops.

Route analysis layer: Of the five feature layers (Stops, Routes, Point Barriers, Line Barriers, and Polygon Barriers), only (located) Stops were used. A minimum of two stops is necessary to create a route.

Route class: It stores the resulting route, or routes, from the analysis.

Shape: The geometry field indicates the geographic location of the network analysis object covers the whole Lebanon. Additional separate analyses cover each mouhafaza. A detailed road map (OpenStreetMaps (OSM) (http://www.openstreetmap.org)) of the capital Beirut is also added.

Location Type: It describes the stop type. Only Stop was used (neither a Waypoint nor a Break). A Stop is a location that the route should visit.

Sequence: As an input field, this number represents the order in which the stops should be visited. The stops are allowed to be reordered by the solver. The optimal sequence is discovered, and Sequence is updated during the solve process. “Change the sequence value is made by dragging stops above or below other stops in the Network Analyst window. The sequence field is specified from the input feature class as the sort field in the load process”.

Options used in the software to process the data, and their justification

Below are some of the options (see the help of the software ArcGIS® 10.6.1) used during the computation of the routes connecting urban centers with shortest path on any of the maps produced.

1. Shapefile of the polygon towns were converted to points located at the center of each polygon using the “feature to point” tool in ArcToolbox® (feature class “polygon input”, with point_location parameter set to CENTROID).

2. A network of squares (a grid with cells of 100X100 m covering the whole Lebanon) was created (Fig. 1) using the “Create Fishnet” tool in ArcToolbox®.

3. Data were stored in a geodatabase file, created by using ArcCatalog®, based on the WGS 84 / UTM zone 36N projection. This type of projection enabled the calculation of the edge lengths (m).

4. To identify the intersection points (see Fig. 2), the grid lines (polylines) were split at their intersection by using “topology” in “ArcCatalog®”. The rule used was “must not intersect”.

5. A network dataset, to be processed by the Network Analyst tool, was built by using ArcCatalog®. Two new files were therefore created, one for the edges (lines) and the other for the junctions (points at intersections).

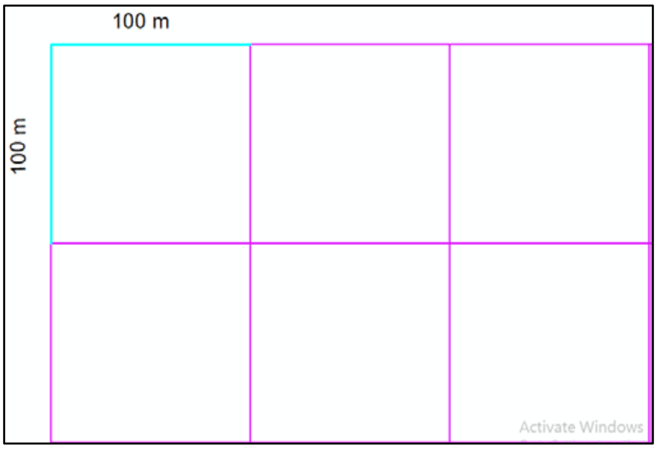

Fig. 1. A diagram showing the construction of a square network, with a grid of 100X100 m cells.

Choice justification of the grid size

For the map of the whole Lebanon, and for those of the single mouhafazat, a network of squares with a grid size having each side of 100 m was used. To test for the optimal grid size (50, 100, 200, or 400 m), a small hypothetical area, with 44 randomly distributed points, was constructed. Larger grid sizes (200 and 400 m) were not possible to solve unless with some intersections, whereas smaller sizes (50 and 100 m) were advantageously solved without intersections and with smaller total distances (Table 1). A 50 m grid size was discarded as it led to an unreasonably long computational time.

Table 1. Relationship between grid size and total distance of the shortest path for the hypothetical example.

Grid size (m)	Total distance (m)	Nr. of Intersections
50	11,547.02	0
100	11,846.95	0
200	11,946.92	1
400	12,296.83	3

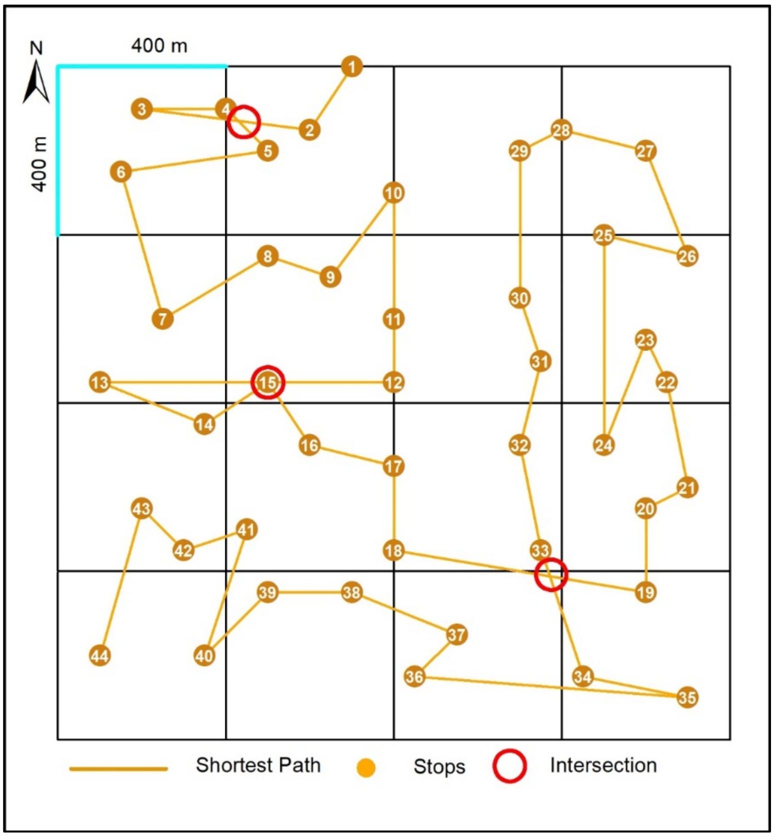

Fig. 2. The convergence of routes takes place within a grid dimension of 400x400m. Urban center locations are based on the same hypothetical example of Table 1.

In the GBA (including the administrative Beirut), the road networks are dense enough to connect any two adjacent districts using possibly more than a single road. The shortest path was therefore produced following only the real road network available, rather than simply choosing a virtual straight-line connection.

In ArcMap®:

1. The junction and edge files were added to the ArcMap®. The network analyst tool was activated, with only the "new route" option being used. The location of towns and villages was loaded.

2. In the layer properties of network analyst:

a. Under "output shape type", "straight line" was selected. The selection of "true shape" was avoided as it would force the route to follow the grid edges. These edges do not necessarily represent actual roads. Running on edges would dramatically increase the probability of having intersections and would lead to an unnecessary increase in the total distance.

b. Under "U-Turns at Junctions", the selected option was "allowed".

c. Under "reorder stops to find optimal route", "preserve first stop" and "preserve last stop" were selected. This facilitates the movement between designated origin and destination points.

d. Under "Layer Properties", the impedance was set as Length (m).

3. “Solve” using Dijkstra’s algorithm.

Hot spot analysis is a spatial analysis, and a mapping technique, that identifies clustering of spatial phenomena. These occurrences are represented as coordinates on a cartographic display, denoting the positions of various incidents or entities. Within the context of this current research, a "hotspot" pertains to a geographic region characterized by a heightened concentration of urban centers in relation to an anticipated count, assuming a stochastic dispersion of said centers. Optimized hotspot analysis (ESRI, 2021) was used to identify the clusters of hot spots and cold spots of the urban centers. The Optimized Hot Spot Analysis tool interrogates the data to obtain the settings that will yield optimal hot spot results. The tool identifies an appropriate scale of analysis.

Results

The use of the Dijkstra's algorithm, with a grid size of 100 x100 m, enabled the tracing of a short path network connecting the Lebanese towns and villages without intersections. Figure 3 shows the shortest path map for the whole Lebanon, with a total distance of 3,313.2 km. The number of urban centers used was 1,568.

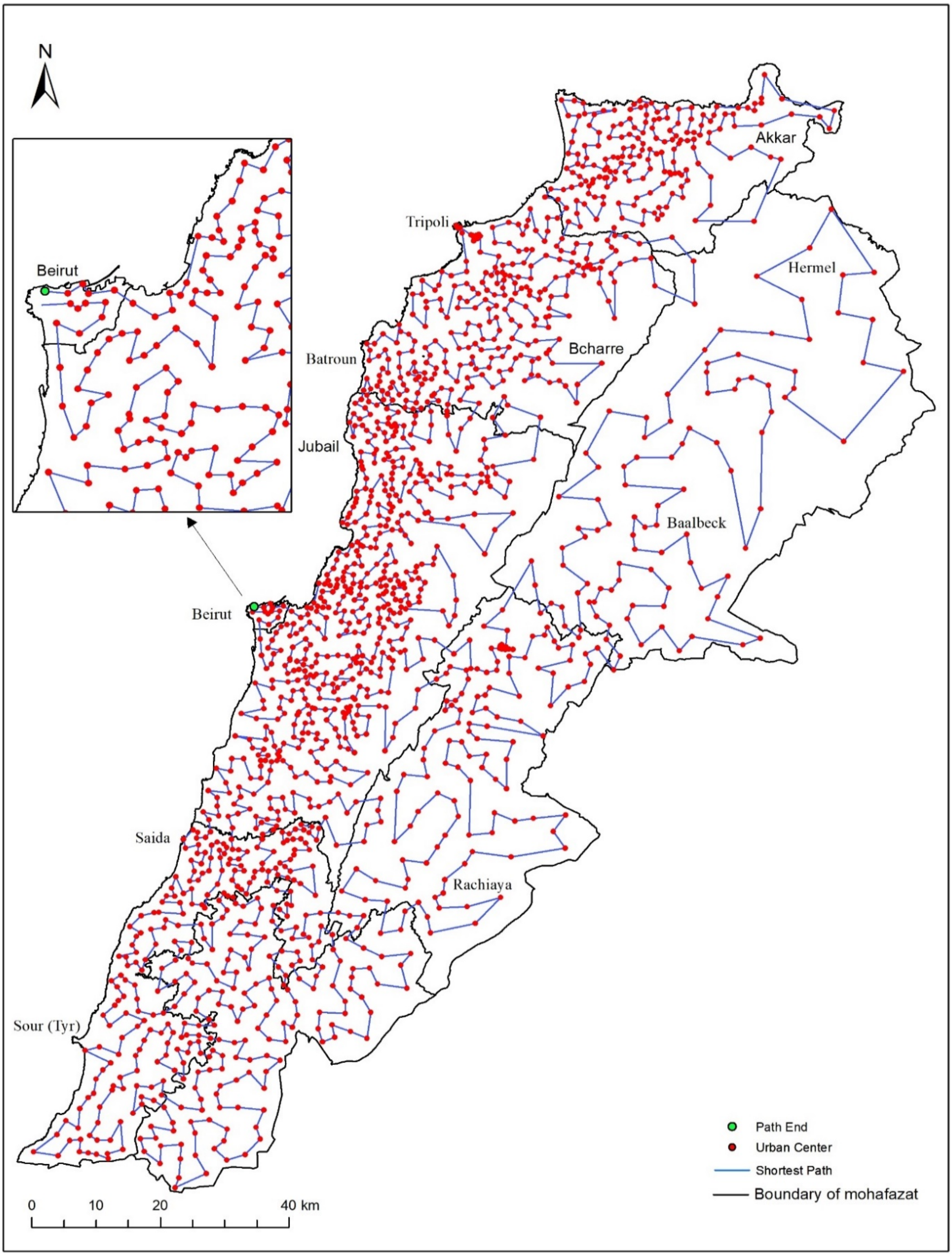

Fig. 3. The shortest path network for the whole Lebanon. Path terminations are chosen at the geographic coordinates of Ras Beyrouth, and the nearest location to it (Ain el-Mreissé).

Figure 4 shows the map produced for the Greater Beirut area. The roads were obtained from OSM to map the shortest path between urban centers of the capital Beirut and its suburbs. Greater Beirut area possesses 31 urban centers. The total distance of this road (shortest path) is 50.2 km.

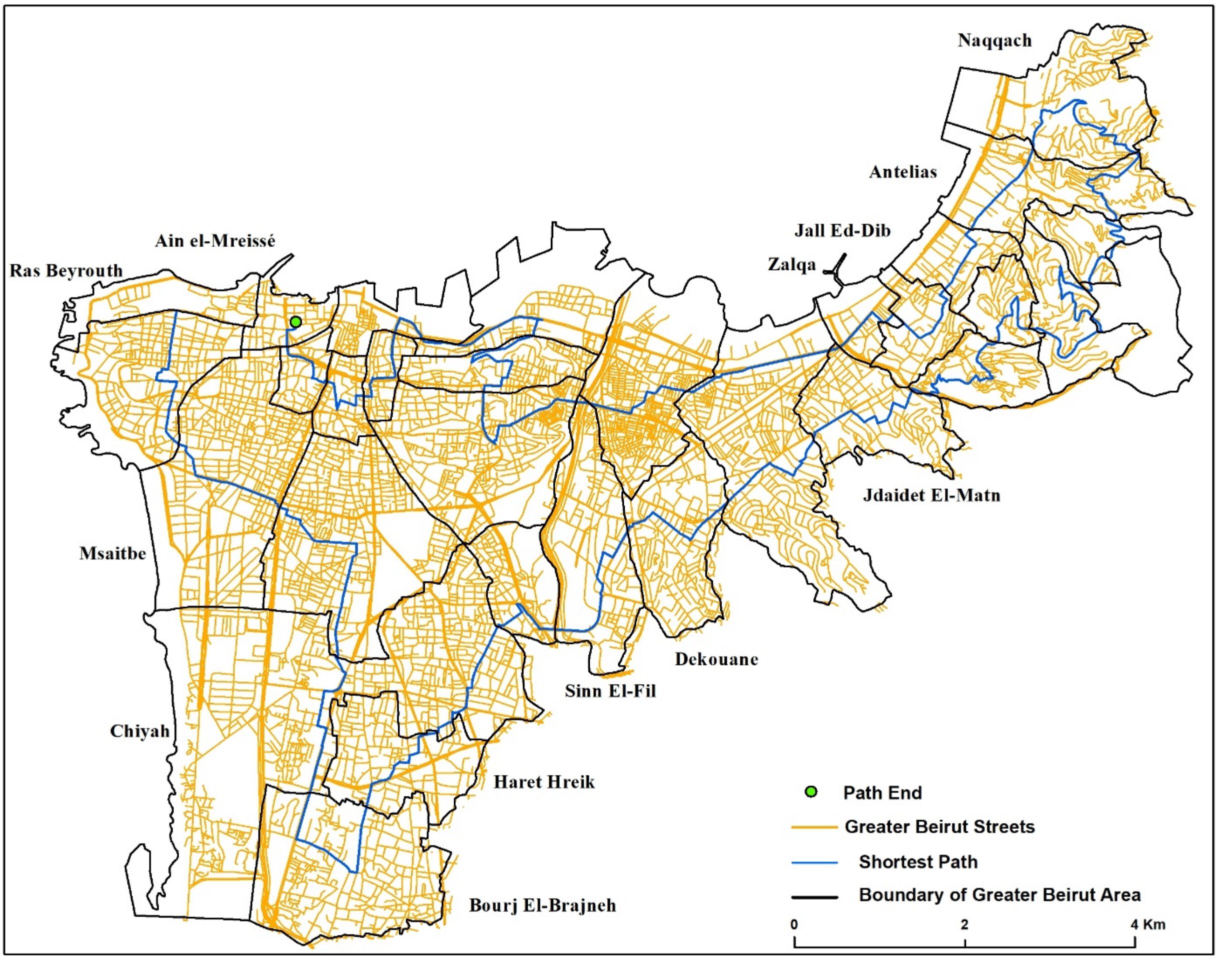

Fig. 4. The optimal route for the Greater Beirut area's shortest path. Path terminations are chosen at the geographic coordinates of Ain el-Mreissé and Minet el-Hosn.

Following is a set of maps showing the shortest paths for each of the 8 Lebanese mouhafazat (Figs. 5 to 11).

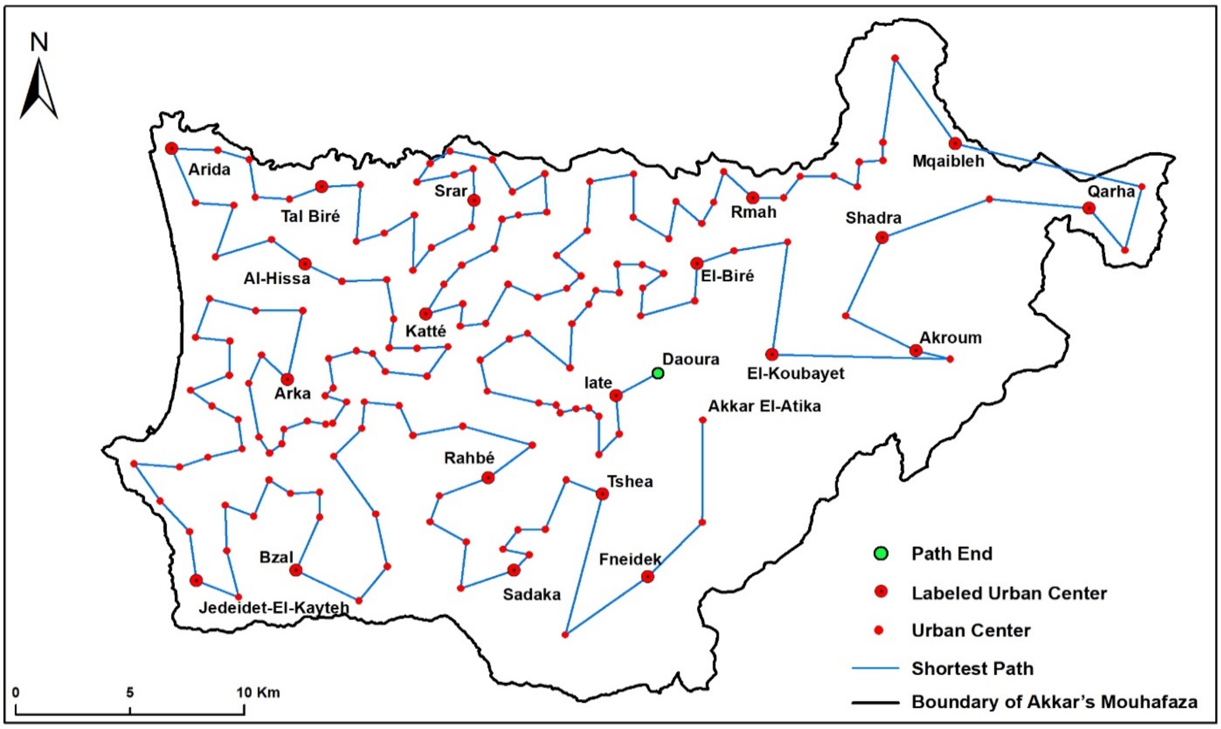

Fig. 5. The optimal route for the Akkar Mouhafaza's shortest path. Path terminations are chosen at the geographic coordinates of Akkar El-Atikaas and Daoura.

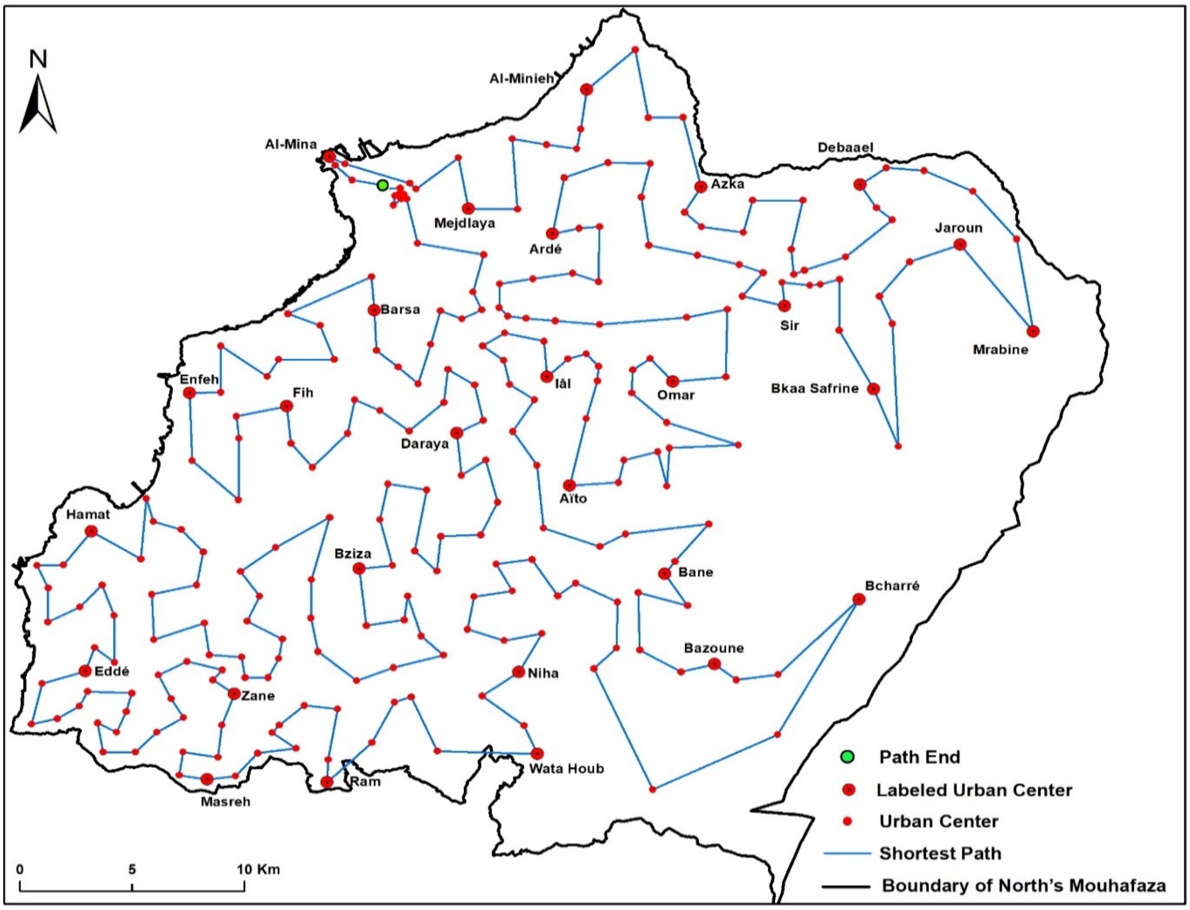

Fig. 6. The optimal route for the North Mouhafaza's shortest path. Path terminations are chosen at the geographic coordinates of Tripoli Jardins and Tripoli Al-Tal.

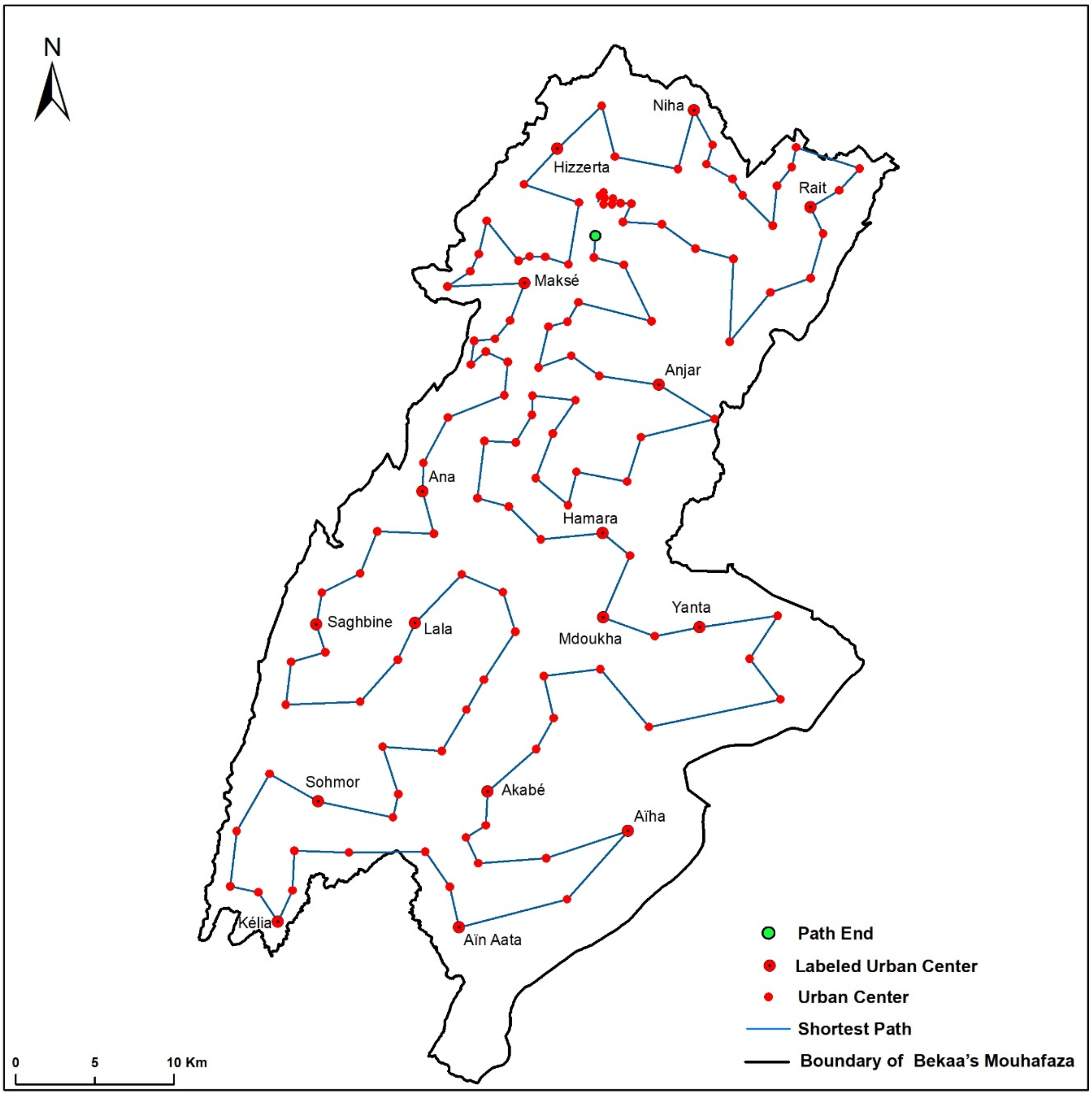

Fig. 7. The optimal route for the Bekaa Mouhafaza's shortest path. Path terminations are chosen at the geographic coordinates of Zahlé Saidet Al-Najda and Ksara.

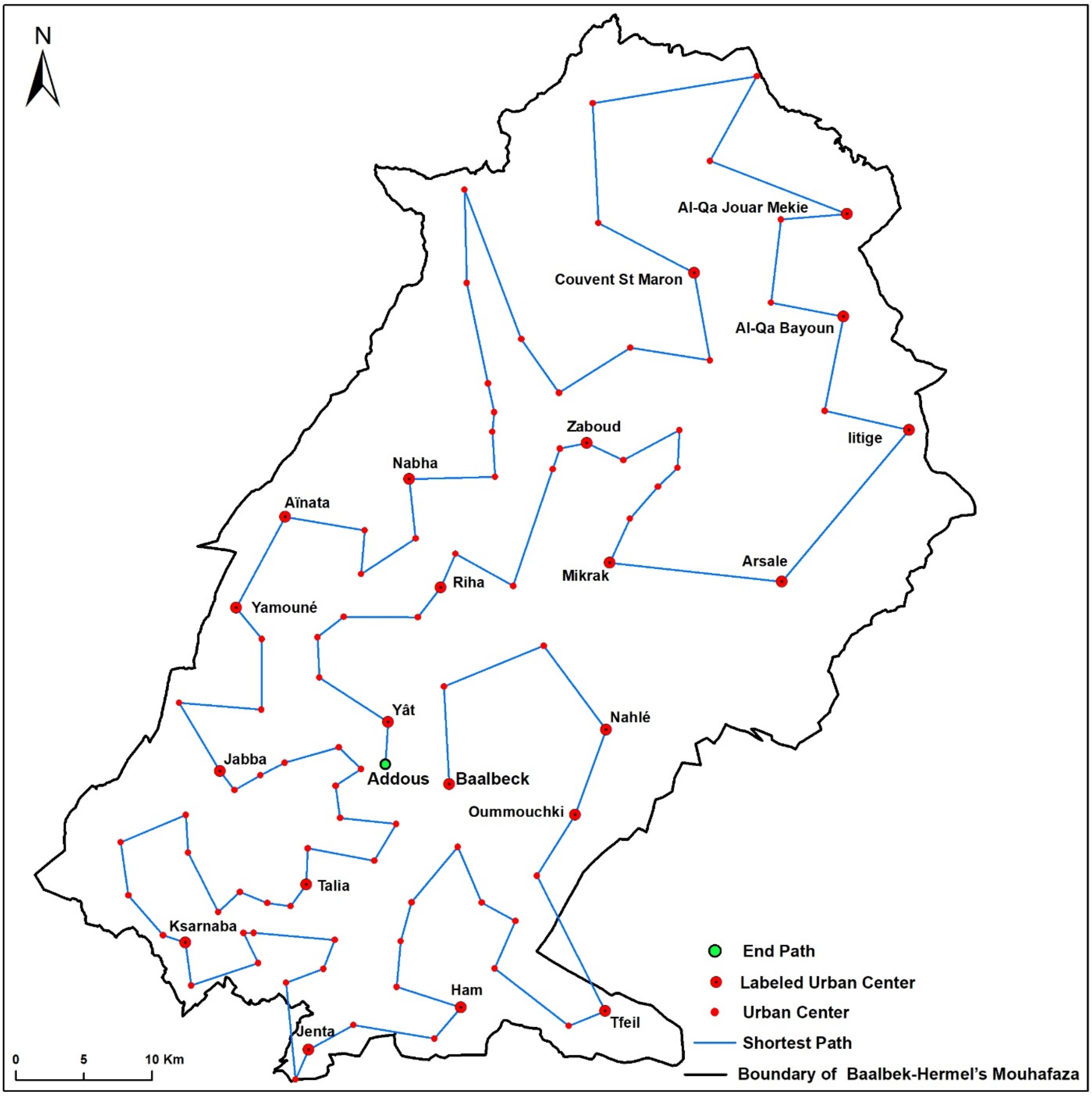

Fig. 8. The optimal route for the Baalbek-Hermel Mouhafaza's shortest path. Path terminations are chosen at the geographic coordinates of Baalbek and Addous.

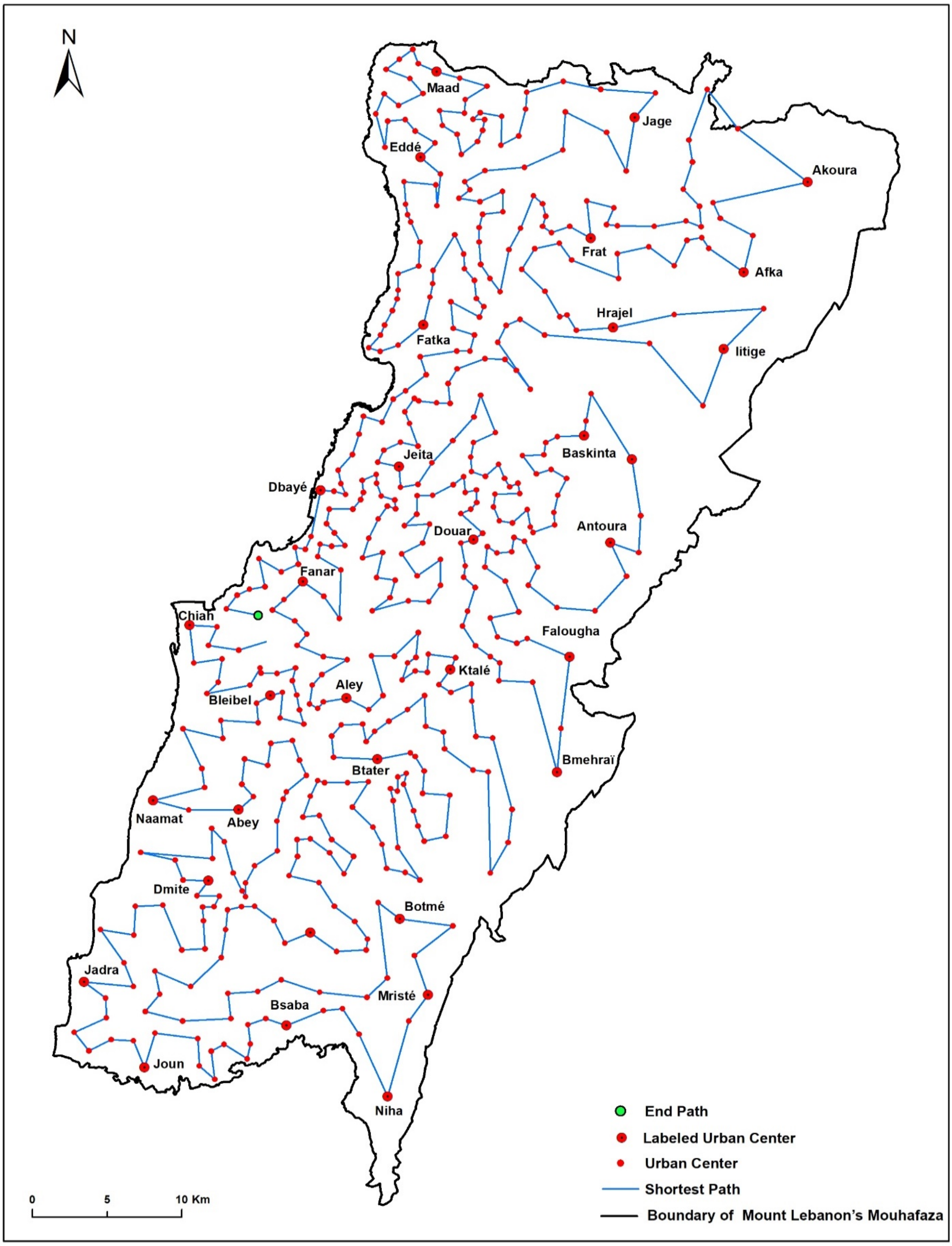

Fig. 9. The optimal route for the Mount Lebanon Mouhafaza's shortest path. Path terminations are chosen at the geographic coordinates of Baabda and Hazmiyeh.

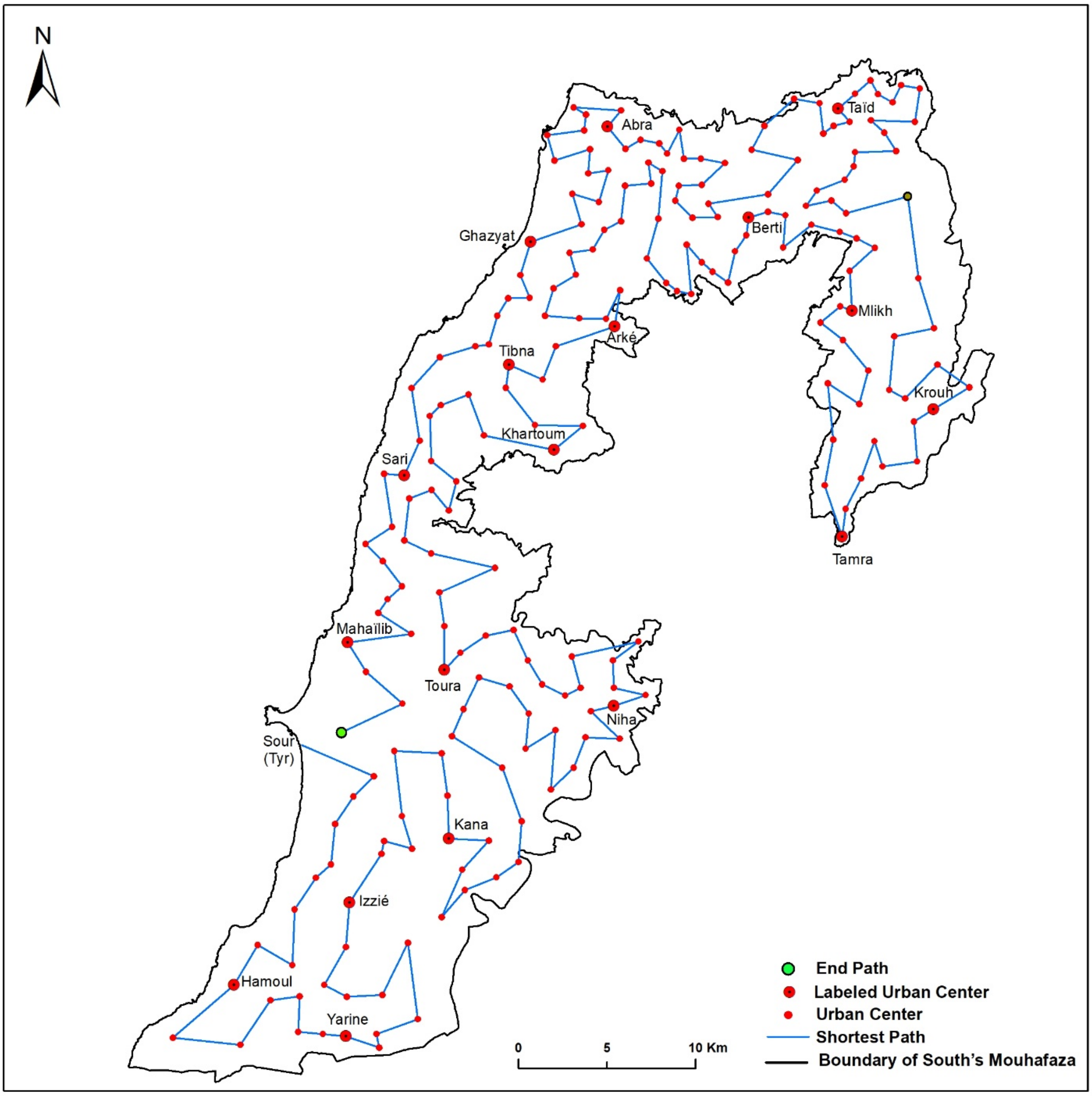

Fig. 10. The optimal route for the South Mouhafaza's shortest path. Path terminations are chosen at the geographic coordinates of Tyre and Borge El-Chémalias.

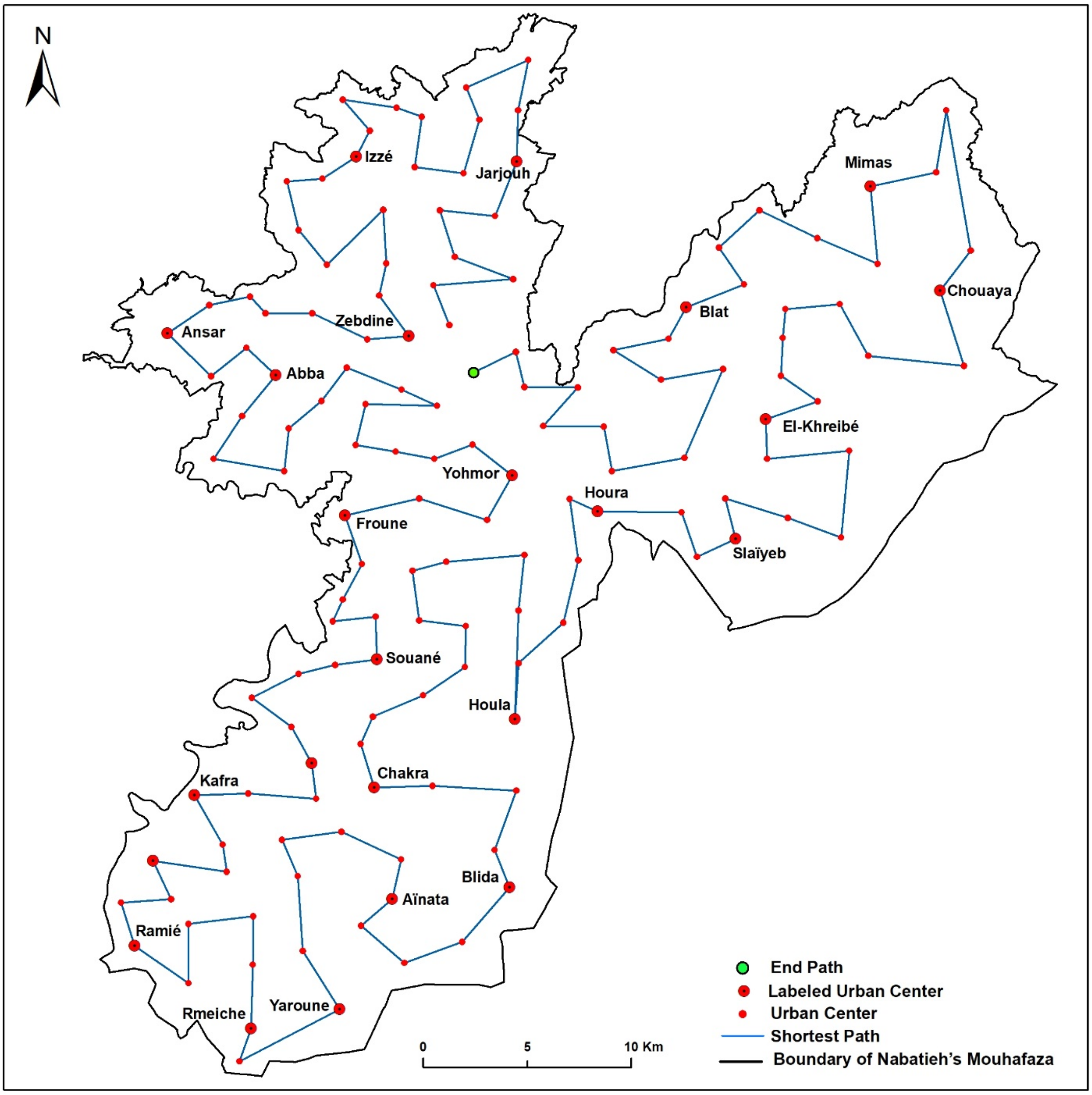

Fig. 11. The optimal route for the Nabatieh Mouhafaza's shortest path. Path terminations are chosen at the geographic coordinates of Nabatiyé El-Tahta and Nabatiyé El-Faouka.

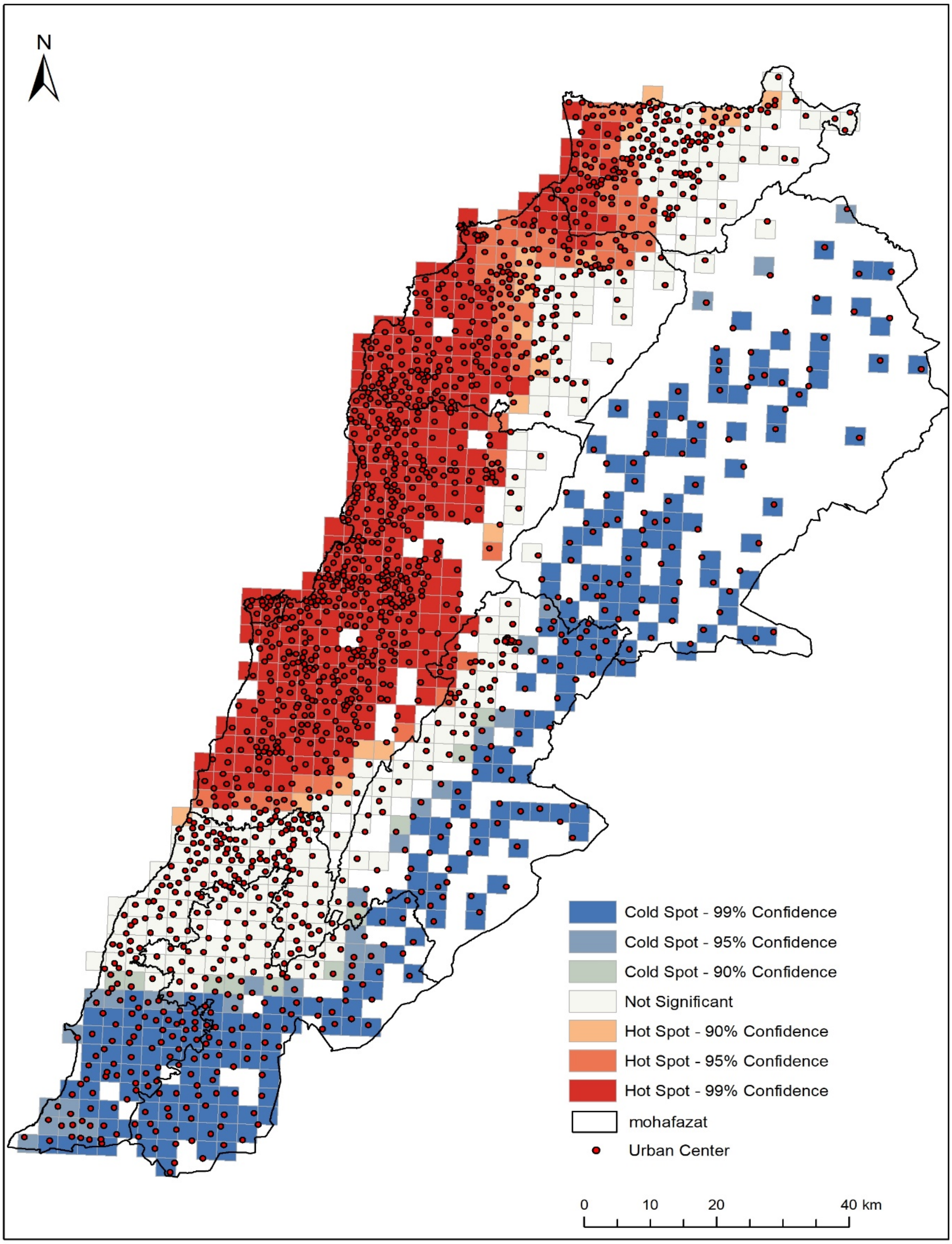

Fig. 12. Optimized Hot Spot Analysis of Lebanese urban centers. It is based on the clustering of the urban centers rather than on the calculated shortest paths.

Table 2. Total and average distances of the shortest path in each mouhafaza.

Mouhafaza	Nr. urban centers	Area (km^2)	Total shortest path (km)	Average distance (km)
Akkar	173	792.16	317.28	1.83
Baalbek - Hermel	98	2,850.85	465.41	4.75
Beirut	13	21.25	14.19	1.09
Bekaa	133	1,405.80	362.54	2.73
Mount Lebanon	514	1,986.51	863.95	1.68
Nabatiyeh	147	1,095.92	380.87	2.59
North	264	1,178.88	480.32	1.82
South	226	916.71	431.02	1.91

The comparison of the total area of the mouhafaza with the average shortest path distance between any two urban centers in it (Table 2) indicates a pseudo-linear trend ($r^2 = 0.66$). The Beirut Mouhafaza has obviously the densest clustering of urban centers, whereas that of Baalbek – Hermel has the sparsest (see also Fig. 12). By contrast, the density of the urban centers per mouhafaza (number of centers per km^2) shows negligible “linearity” ($r^2 = 0.08$). Mount Lebanon Mouhafaza shows the longest total shortest path.

Discussion and conclusions

Shortest path maps play a pivotal role in delivering solutions to a diverse range of spatial challenges. Dijkstra’s algorithm for shortest path was used to connect all the Lebanese towns and villages without any intersection. This provides maps with shortest paths for non-intersecting networks covering Lebanon, either totally or for each mouhafaza.

The smaller the grid, the shorter the distance. The possible reason for the change of the total distance with the grid size is that the spatial details inside any polygon representing an urban center appear with a different resolution with the changing scale at which these details are observed (change of the fractal dimension). The total distance changes not only by a more precise location of the urban center (given the higher resolution), but it is mainly due to a different order of the points (representing the urban centers) through which the path passes.

In light of the computational time constraints applicable to exceedingly small grid dimensions, the prevailing terminology "shortest path" effectively conveys the concept of "a path of acceptable brevity" within the context of grid utilization. “Shortest path” still bears its full meaning when real roads are followed.

The average shortest path between the urban centers varies from one mouhafaza to another by a factor about 4. The highest being recorded for that of Baalbek – Hermel with 4.75 km (sparsest clustering of urban centers), and the lowest for Beirut (1.09 km). Neglecting the topographic differences and the difference of traffic speed between the different mouhafazat (due *e.g.*, to road quality), a higher average shortest path between the urban centers leads to a higher cost of fuel consumption in vehicles. Obviously, the concentration of pollution in the atmosphere given by vehicle traffic is also a function of a variety of factors (climate …).

The connection between any two points resulted in a straight line as it represents the shortest path, even though this line is adapted to the local conditions (topography, urbanization, archeology …) in the real world.

Although a typical application of the networks built is to minimize the drone path (or minimize the total distance of power grid lines …), this method does not consider topographic characteristics (altitude, slope) and land cover (urban areas, waterways, lakes). In relatively flat areas (*e.g.*, administrative Beirut), the factor of altitude difference, and thus its contribution to distance increase, is less pronounced. Nevertheless, there exists a greater disparity in elevation in numerous other instances. East Lebanon is isolated from its west in both the north and the center, due to the extension of the Lebanon Mountains range. On the other hand, in the south of Lebanon, the south-east is connected to the west over several transects, with minor topographic differences.

The total Lebanese area according to its constitution is 10,452 km^2, while the area on the loaded map was 10,248 km^2. This discrepancy may in part be due to the fractal dimension used.

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